OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4733

Probability & Statistics 2

Wednesday **25 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 In a study of urban foxes it is found that on average there are 2 foxes in every 3 acres.

(i) Use a Poisson distribution to find the probability that, at a given moment,

(a) in a randomly chosen area of 3 acres there are at least 4 foxes, [2]

(b) in a randomly chosen area of 1 acre there are exactly 2 foxes. [3]

(ii) Explain briefly why a Poisson distribution might not be a suitable model. [2]

2 The random variable W has the distribution $\mathrm{B}(40, \frac{2}{7})$. Use an appropriate approximation to find $\mathrm{P}(W > 13)$. [7]

3 The manufacturers of a brand of chocolates claim that, on average, 30% of their chocolates have hard centres. In a random sample of 8 chocolates from this manufacturer, 5 had hard centres. Test, at the 5% significance level, whether there is evidence that the population proportion of chocolates with hard centres is not 30%, stating your hypotheses clearly. Show the values of any relevant probabilities. [7]

4 DVD players are tested after manufacture. The probability that a randomly chosen DVD player is defective is 0.02. The number of defective players in a random sample of size 80 is denoted by R.

(i) Use an appropriate approximation to find $\mathrm{P}(R \geqslant 2)$. [4]

(ii) Find the smallest value of r for which $\mathrm{P}(R \geqslant r) < 0.01$. [3]

5 In an investment model the increase, $Y\%$, in the value of an investment in one year is modelled as a continuous random variable with the distribution $\mathrm{N}(\mu, \frac{1}{4}\mu^2)$. The value of μ depends on the type of investment chosen.

(i) Find $\mathrm{P}(Y < 0)$, showing that it is independent of the value of μ. [4]

(ii) Given that $\mu = 6$, find the probability that $Y < 9$ in each of three randomly chosen years. [4]

(iii) Explain why the calculation in part (ii) might not be valid if applied to three consecutive years. [1]

6 Alex obtained the actual waist measurements, w inches, of a random sample of 50 pairs of jeans, each of which was labelled as having a 32-inch waist. The results are summarised by

$$n = 50, \quad \Sigma w = 1615.0, \quad \Sigma w^2 = 52\,214.50.$$

Test, at the 0.1% significance level, whether this sample provides evidence that the mean waist measurement of jeans labelled as having 32-inch waists is in fact greater than 32 inches. State your hypotheses clearly. [10]

7 The random variable X has the distribution $N(\mu, 8^2)$. The mean of a random sample of 12 observations of X is denoted by $\overline{X}$. A test is carried out at the 1% significance level of the null hypothesis H_0: $\mu = 80$ against the alternative hypothesis H_1: $\mu < 80$. The test is summarised as follows: 'Reject H_0 if $\overline{X} < c$; otherwise do not reject H_0'.

(i) Calculate the value of c. [4]

(ii) Assuming that $\mu = 80$, state whether the conclusion of the test is correct, results in a Type I error, or results in a Type II error if:

(a) $\overline{X} = 74.0$, [1]

(b) $\overline{X} = 75.0$. [1]

(iii) Independent repetitions of the above test, using the value of c found in part **(i)**, suggest that in fact the probability of rejecting the null hypothesis is 0.06. Use this information to calculate the value of μ. [4]

8 A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where n and k are positive constants.

(i) Find k in terms of n. [3]

(ii) Show that $E(X) = \dfrac{n+1}{n+2}$. [3]

It is given that $n = 3$.

(iii) Find the variance of X. [3]

(iv) One hundred observations of X are taken, and the mean of the observations is denoted by $\overline{X}$. Write down the approximate distribution of $\overline{X}$, giving the values of any parameters. [3]

(v) Write down the mean and the variance of the random variable Y with probability density function given by

$$g(y) = \begin{cases} 4\left(y + \frac{4}{5}\right)^3 & -\frac{4}{5} \leqslant y \leqslant \frac{1}{5}, \\ 0 & \text{otherwise.} \end{cases}$$ [3]